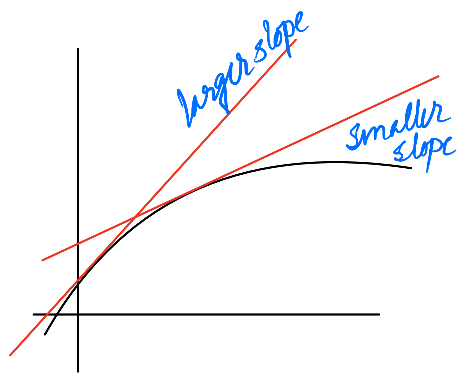
Recall: $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Note: We can take derivative as a function

$a \rightsquigarrow \boxed{\text{Derivative}} \rightsquigarrow$ Slope of the tangent line to the function we are dealing with at $x = a$

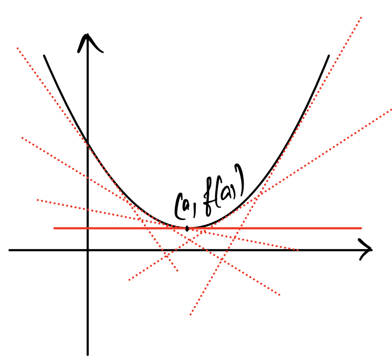
Notation of derivatives of $y = f(x)$:

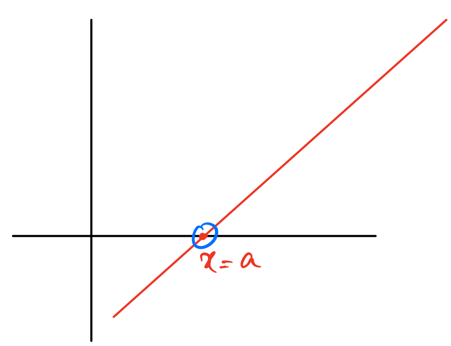
- $f'(x)$
- $\frac{d}{dx}(f(x))$
- y'
- $\frac{dy}{dx} = \frac{d}{dx}(y)$



Graphing Derivatives :-

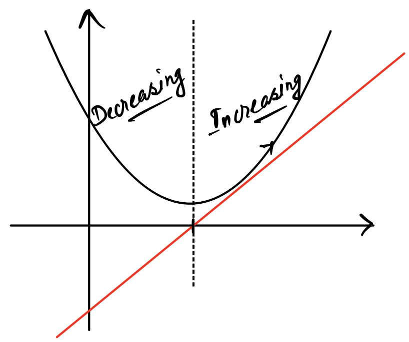
① $f'(x) = 0$ or Horizontal Tangent.





If $f'(x)$ is 0 for some $x = a$, then the function $f(x)$ changes it's direction at $x = a$.

② $f'(x) > 0$ or Increasing function:



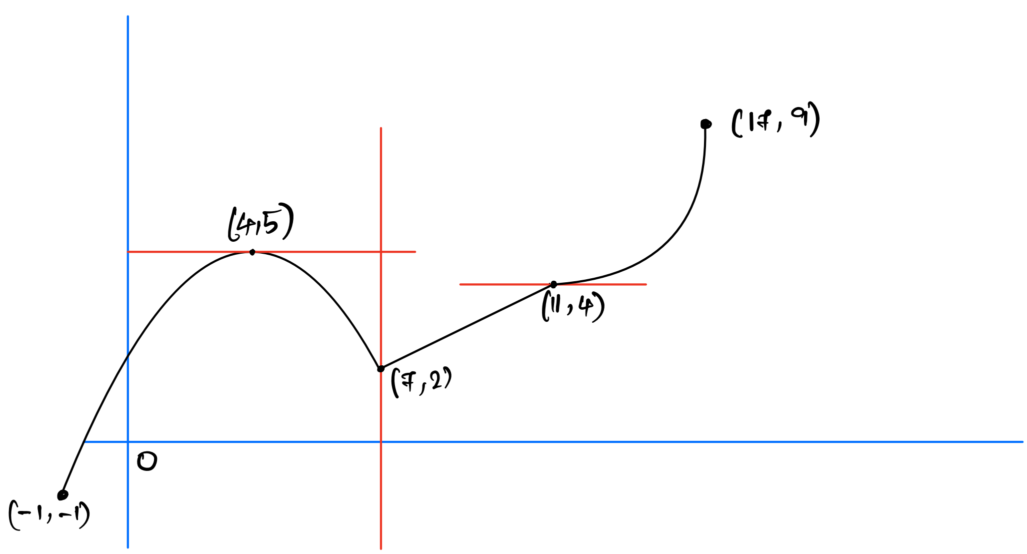
③ $f'(x) < 0$ or Decreasing function.

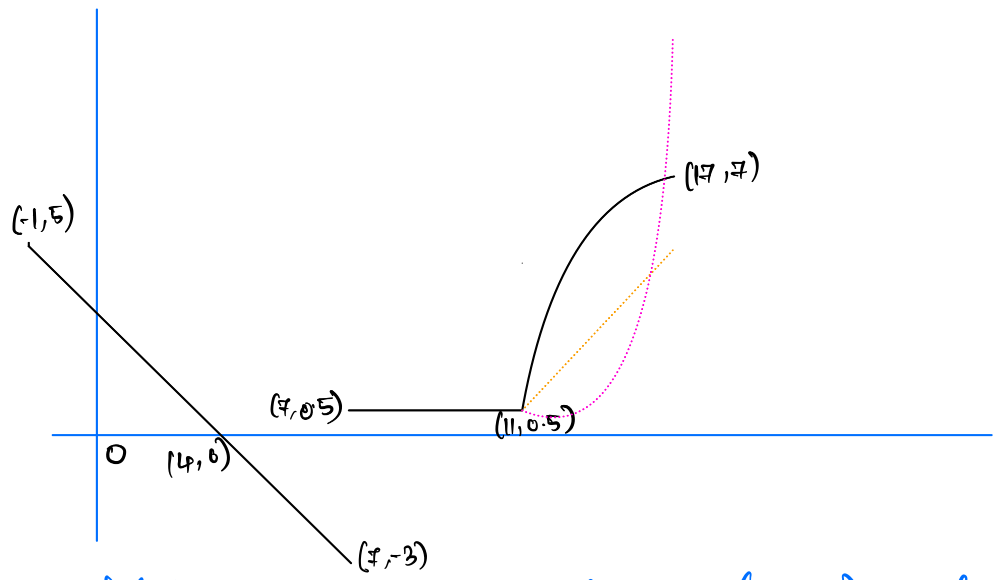
④ $f'(x)$ is undefined: Vertical

As. $f'(x) = \dfrac{\text{change in } y \text{ at some pt}}{\text{change in } x \text{ at the same pt}}$

$f'(x)$ undefined means no change in x-coord.

Try: Graph $f'(x)$, where $f(x)$ is given as follows





Note:

- $f(x)$ is increasing between $(-1,-1)$ & $(4,5)$ in a non-linear path (seems like a parabola)

  $\Rightarrow f'(x) > 0$, x in $(-1,4)$ & $f'(x)$ is not constant

- $f(x)$ has a horizontal tangent at $(4,5)$ point.

- $f(x)$ is decreasing between $(4,5)$ & $(7,2)$ in a non-linear path (seems like the same parabola)

  $\Rightarrow f'(x) < 0$, x in $(4,7)$ & $f'(x)$ is not constant

- Between $(7,2)$ & $(11,4)$, $f(x)$ is a straight line with fixed slope $= \frac{4-2}{11-7} = \frac{2}{4} = 0.5$

- Between $(11,4)$ & $(17,9)$ we have an increasing non-linear (possibly non-parabolic curve)

  $\Rightarrow f'(x) > 0$ in $(11,17)$, $f'(x)$ is not constant, and possibly not a st. line path (being possibly non-parabolic) ie, any positive curve between $x=11$ & $x=17$ will work out (possibly st. line also).